LHC Project Note 269

October 24, 2001

Jean-Pierre.Koutchouk@cern.ch

Requirements for the Measurement of the Non-Linear Beam Motion in LHC

J. P. Koutchouk

Keywords: Beam measurements, Instrumentation, Optics

Summary

The respective perturbations of the beam transverse spectrum by either the non-linear beam motion or the non-linear transfer function of the beam position monitors (BPM) are compared. A level of significance of the perturbation is defined assuming a perfect LHC with lattice sextupoles only. The results show a large robustness against the BPM non-linearity. This allows to relax significantly the requirement of the non-linearity of the BPM system.

1 Introduction

The most direct method to measure the non-linearity of the beam motion is to kick the beam transversely and observe its trajectory with beam position monitors (BPM). The significant non-linearities in LHC are essentially b_5, a_4, b_4 and b_6, i.e. decrease with at least the third power of the amplitude. Their effect is by design negligible for amplitudes corresponding to the beam distribution ($\pm 3\sigma$). The measurement of the non-linearity requires thus large amplitudes but not so large as risking to quench the machine (the aperture without collimators is around 10σ). A good compromise seems to kick to amplitudes in the range of 5 to 7σ (about 7 mm at 450 GeV). In this amplitude range, the linearity of the BPM transfer function shall be such as to avoid fake non-linear signals which could be mis-interpreted. The non-linearity of the BPM transfer function arises from the geometry of the sensors and from the normalization and processing of the analog signals. The systematic part of the non-linearity is corrected either electronically or by data post-processing (this will be the case in LHC, was the case in the LEP BOM system but not in the SPS one). After correction, a residual non-linearity, different from prototype to prototype, is observed. The aim of this note is to specify a tolerance based on beam dynamics requirements for this residual random non-linearity.

2 Methodology

In the LHC regime of weak non-linear perturbations, their effect appears mostly in the Fourier transform of the beam trajectory as a set of 'harmonics'. There is presently no identified criterion to define bounds for these harmonics which would yield 'good' beam conditions (lifetime, background,...). Attempts already made at specifying the BPM non-linearity gave tight requirements with expected significant consequences on the complexity/cost of the BPM system. They are briefly summarized.

In [1], the authors specified a tight peak non-linear error ($\pm 50\mu$m) using a direct comparison between the BPM non-linearity and resolution in the space domain. The reference beam oscillation amplitude was taken to be 1σ. As noted above, the frequency domain is the most relevant one. In this domain, the spectral characteristics of the resolution (additive noise) and of the non-linearity of the transfer function (multiplicative noise) are very different and do not lend themselves to a direct comparison. The reference beam amplitude was furthermore taken small, thereby tightening the requirements.

The non-linearity of the BPM transfer function is mostly of third order in the amplitude and produces an octupole-like perturbation. In [2], this perturbation was required to be smaller than the spectral line of the resonance (4,0) by parasitic octupoles. The latter had been studied in the search of a correlation of the dynamic aperture with systematic low-order resonances and found insignificant for the beam dynamics [3]. This is a consequence of the mixing due to the rapid rotation of the resonance phase even when the cell phase advance was closer to 90° (before the tune split). Several studies on later versions of LHC confirm the negligible excitation of (4,0), e.g. [4], [5], [7]. In fact the octupoles act mostly through the amplitude detuning. This term of comparison does not seem appropriate.

Finally the tolerance on the non-linearity was estimated [8] using the method of measurement of the resonance driving terms discussed in [9]. It is however not possible to disentangle the effect of the BPM errors proper from the noise sensitivity of this specific method.

In the light of these difficulties, we have revisited the issue in an attempt to identify conditions which would be both sufficient and necessary. The approach is simple enough to lend itself to check and discussion:

- The perturbation of the beam motion or of the BPM's are quantified in the frequency domain only.
- The spectral information is not processed. Simply a norm of the perturbation is defined. It is taken to be the largest amplitude of the harmonics excited by the non-linearities. In some cases another norm (the quadratic sum of the first few harmonics) was tested and gave very similar results.
- A significance level of the non-linearity is defined by computing the norm of the spectrum in a case where the non-linearity is deemed negligible. We considered a 'perfect' LHC only perturbed by the chromaticity sextupoles in a detuned lattice.
- Norms are computed for realistically perturbed LHC's and perfect BPM's. This is used to verify that the significance level defined above is small enough as compared to the expected dynamic range of the non-linear perturbation.

- The norm of the spectrum is calculated again in the case of a perfectly linear beam motion and a non-linear BPM transfer function. The tolerance is set by requiring this norm to be smaller than the significance level.

In all cases, the amplitude scale of the spectra is normalized to the amplitude of the main betatron line.

3 The Model used for the Calculations

The LHC model used for this study was the latest available in the database at the moment (end 2000), i.e. v6.-2 with the error table 9607. Whenever a perturbed LHC is necessary, the default seed 55 is used, known as a 'typical' LHC. Linear imperfections are absent in such a way that the harmonics are only due to the non-linear fields and not, e.g. to linear coupling excited by skew gradients. The lattice version 6.-2 is identical to version 6.0 and differs only by insignificant details from the latest versions. The optics of v6.-2 is slightly different from that of v6.0 due to the tune split of 4 (now 5). This change is taken to be insignificant for our purpose. The error table 9607, widely used, includes stronger multipoles than the latest operational version 9901. This does not seem critical for this analysis, as it is only used for the estimate of the dynamic range of the spectrum perturbation.

Tracking is performed with MAD8 using three initial conditions in the transverse plane x, y: mostly horizontal motion (15 degrees), at 45 degrees and mostly vertical motion (75 degrees). The largest norm of the three spectra is retained. The tracking is carried out over a typical observation period of 1000 turns (optimistic in practice: the decoherence may only allow a few hundred turns). This is only a few synchrotron periods and 4D tracking was used. The harmonic content of the spectrum of the motion is therefore slightly less rich, in the absence of feed-down of e.g. b_5 with the off-momentum orbit. This seems safe for our goal which is to specify the non-linearity of the BPM's to be smaller than that of the beam.

4 Significance Level of the Non-Linear Perturbation

We take as an axiom that the effect of the lattice sextupoles in the perfect LHC lattice on the detuned optics at injection perturbs in a negligible way the beam dynamics. Dynamic aperture calculations and exact scaling show indeed that the dynamic aperture is about 28σ. At collision energy after β-squeeze, it even reaches about 70σ [10] thanks to the adiabatic shrinking of the emittance, in spite of the stronger sextupoles. This hypothesis seems confirmed by the experience that normal conducting storage rings are easy to handle at injection from the point of view of single beam dynamics.

The norm of the harmonics in these conditions are given in table 1. The largest harmonic corresponds to the third order resonance (1,2) but the (3,0) is only slightly less excited. We conclude that the significance level for the non-linear motion is at least 3% of the main peak.

5 Dynamic Range of the Non-Linear Perturbation in a realistic LHC

To be consistent with the LHC design assumptions, we have to take into account the safety factor of 2 implemented in the dynamic aperture studies. The distortion of the motion at 5σ in the real machine is deemed to be modelled by a 10σ oscillation in the tracking model. We have tracked one instance of a perturbed LHC at injection, namely seed 55 which represents an 'average' LHC. Several cases were considered differing by the betatron tunes and a 'typical' fault. The betatron tunes were varied in a reasonable range to change the sensitivity to resonances. The fault was such as to enhance the sensitivity to the 3rd order resonances, close to the working point:

- (1) the nominal perturbed machine seed 55,
- the same machine with other betatron tunes: (2) .31/.32, (3) .29/.30, (4) .27/.32,
- (5) the nominal perturbed machine seed 55 where the spool piece circuit b_3 in arc 12 is faulty and the resulting chromaticity corrected by the lattice sextupoles.

Case		*Amplitude*	
		1σ	5σ
Perfect LHC & BPM's	Injection	0.7	3.5
	Collision	0.6	3.1
Perturbed LHC & Perfect BPM's	(1)		20
	(2)		100
	(3)		100
	(4)		12
	(5)		40
Perfect LHC & Imperfect BPM's	$\pm 1500\,\mu$m		1
	$\pm 500\,\mu$m	0.07	0.3
	$\pm 250\,\mu$m	0.04	0.2

Table 1: Norm of the harmonics of the transverse spectrum

The results in table 1 show that a resolution of a few percent of the main peak is indeed a small fraction of the dynamic range.

6 Fake Harmonics due to a Non-Linear Transfer Function of the BPM's

The dominant perturbation of the LHC BPM transfer function is third-order in the measured position [11]. The largest part is corrected hardware-wise in the BPM normalizer. The hypothesis made is that the residual is still mostly third-order in the measured position. This is grosso-modo observed in the first few prototypes and is assumed in the following.

The on-line calibration system of the BPM enforces an exact transfer function at three beam amplitudes: -A, 0, +A, where A is the BPM aperture (about 24 mm). The transfer function after calibration is therefore given by:

$$x_{measured} = x_{true} + \alpha x_{true}^3 - \alpha A^2 x_{true} \qquad (1)$$

The peak deviation from linearity $\hat{\Delta}$ is related to the third-order coefficient α by:

$$\hat{\Delta} = \frac{2}{3\sqrt{3}} \alpha A^3 \qquad (2)$$

The deviation from linearity after calibration for $\hat{\Delta} = 250\mu$m is shown on figure 1. Assuming

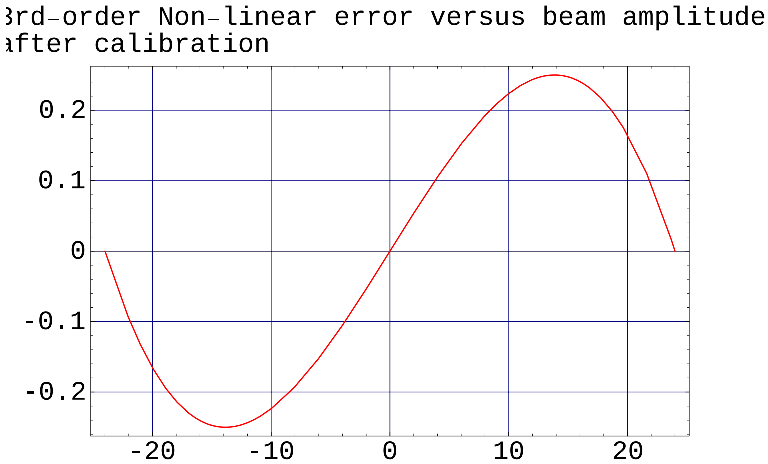

Figure 1: Deviation from linearity in the BPM transfer function versus beam amplitude in mm

a perfectly linear beam motion, the effect of the non-linearity of the transfer function is studied again by computing the norm of the harmonics of the Fourier transform of the measured motion. The results are summarized in table 1. In all cases, the closed orbit is assumed to be at its maximum (4 mm), thereby yielding the worst case for the non-linear distortion.

It turns out that a non-linear perturbation of the BPM transfer function by as much as $\pm$ 1500 μm produces a norm of the harmonics well below the significance level identified in section 4. A fifth-order perturbation would produce an even weaker perturbation as the peak distortion occurs at a larger beam amplitude.

7 Conclusion and Specification

The level of significance of the distortion of the transverse spectrum is established by tracking a perfect LHC, only perturbed by the lattice sextupoles. Table 1 shows clearly that the fake harmonics produced by the non-linearity of the BPM transfer function are still well below significance for distortions of the transfer function as large as $\pm$ 1500 μm (over its full $\pm$ 24 mm range). The robustness stems from the multiplicative nature of this non-linear 'noise'. This is less demanding that the former requirements by a factor 30 or more. The BPM technology allows in fact a significantly better linearity than the quoted requirement from beam dynamics. We therefore tighten the specification of the non-linearity to $\pm$ 500 μm over the full amplitude range. At the present stage of design, it seems that the LHC BPM system will exceed possibly significantly this requirement [12].

References

[1] O. Brüning, W. Herr, F. Ruggiero, F. Schmidt, 'Requirements for the LHC BPM System based on beam dynamics, Memo to H. Schmickler, 22 May 2000 and 6th Meeting of the SPS-LHC Technical Committee, 24th may 2000.

[2] F. Ruggiero, 'Instrumentation Needs for Accelerator Physics', 9th LHC Machine Adv. Committee, 13/11/2000.

[3] Y. Cai, J.P. Koutchouk, 'Correlations of Dynamic Aperture and Resonance Strengths for the LHC v4.3', LHC Project Note 75, 1996.

[4] R. Talman, 'Tune Optimization for Maximum Dynamic Acceptance,II', LHC project report 233, 1998.

[5] L. Jin, Y. Papaphilippou, F. Schmidt, 'Improvement of LHC Dynamic Aperture via Octupole Spool Pieces for the Nominal Tunes', LHC Project Report 253, 1998.

[6] Y. Papaphilippou, F. Schmidt, 'Normal Form Approaches and Resonance Analysis of LHC Models', LHC Project Report 255, 1998.

[7] S. Fartoukh, 'Study of an Alternative Octupole Correction Scheme', unpublished, 1999.

[8] R. Tomas, 'Direct Measurement of Resonant Driving terms', AP Forum 14th Feb. 2001.

[9] F. Bartolini, F. Schmidt, Part. Accel. 59,93(1998).

[10] J.P. Koutchouk *et al.*, 'The LHC Dynamic Aperture', LHC Project Report 296, 1999.

[11] D. Cocq,'The Wide-band Normalizer, a new circuit to measure transverse bunch position in accelerators and colliders', Nucl. Instr. and Methods in Phys. Res. A 416(1998)1-8.

[12] D. Cocq, private communication, Sept. 2001